OPERATION MODES OF WIND POWER PLANTS WITH ASYNCHRONOUS GENERATORS UNDER VARIABLE–FREQUENCY CONTROL

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Mathematical model (model of state) is presented and different operation models of WPPs, containing asynchronous generators, frequency-controlled both from stator side and from rotor side have been studied on this model. Analytical calculation method has been offered, and its comparison with presented mathematical model has been carried out.

Key words: Wind power plant, asynchronous generators, frequency control, generator stator, generator motor, operation mode.

Last years wind power engineering has rapid growth. Today wind power plants of up to 5 MW unit power are operating in parallel with electric power network. In 2005 the total capacity of wind power plants in the world constituted 58 GW.

The low-speed synchronous generators, squirrel-cage motors, double-fed asynchronous machines find an application as electromechanical converters for wind power plants (WPP).

To raise the WPPs' operating efficiency within a specific range of wind speed variations, it needs to adjust also accordingly the rotational speed of wind motor (WM) and of jointed generator [1,2,3]. Under such control the wind speed utilization factor reaches the maximum values for corresponding wind speed values, thus the electric power output increases.

The expressions of wind motor's capacity and torque are accordingly recorded in the form of [1]:

$$P_{wm} = \frac{1}{2} \rho \pi R^2 \cdot V^3 \cdot c_p \tag{1}$$

$$M_{wm} = \frac{1}{2} \rho \pi R^3 \cdot V^2 \cdot \mu, \qquad (2)$$

where ρ – air mass density, R – wind wheel radius, V – wind speed; c_p – wind power utilization factor, μ – relative wind motor's motive torque.

The interrelation between wind power utilization factor c_p and relative motive torque μ is determined on the correlation:

$$\mathbf{c}_{\mathbf{p}} = \boldsymbol{\mu} \cdot \mathbf{Z} \,, \tag{3}$$

where $-Z = \frac{\omega_{wm}R}{V}$ modules number or specific speed, ω_{wm} – wind motor's angular rotational speed.

If to determine μ from the expression (3) and substitute it in expression (2) with taking into account the dependence of specific speed on ω_{wm} and V, that, naturally, we shall obtain:

$$M_{wm} = \frac{P_{wm}}{\omega_{wm}}.$$
 (4)

Provided wind motor's rotational speed is controlled in proportion to wind speed, the number of modules Z must theoretically remain constant i.e.

$$Z = Z_{opt} = \frac{\omega_{wm} \cdot R}{V} = \text{const}.$$
 (5)

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On WM's aerodynamic characteristic Z_{opt} , c_p and μ must also remain constant, but in real wind power plants it is impossible to achieve this exactly.

For instance, for WPP of Gamesa G–52 type of 850 KW power utilizing the double-fed asynchronous machine in the capacity of an electromechanical converter, the WM's rotational speed changes from 30,8 rev./min. (0,513 rev./s) to 14,6 rev./min. (0,24 rev./s), i.e. wind motor's rotational speed is adjusted from 0,513 rev./s to 0,24 rev./s (the depth of change is 0,513/0,24 =2,1) within the range of wind speed variation from 10,45 m/s to 4,96 m/s (the depth of change is 10,45/4,96=2,1). For considered wind motor the wind power utilization factor c_p changes within the indicated range from $c_{p min} = 0,390$ to $c_{p max}=0,452$.

However, if, within the control range of WM's rotational speed, to operate with factor's average value c_{pav} , which ought to be defined as arithmetical mean value on expression:

$$c_{p av} = \frac{\sum_{i=1}^{n} c_{pi}}{n},$$
 (6)

where $-c_{pi}$ – is possible fixed values of wind speed utilization factor c_p within control range, n – is a number of fixed values, then the power computational error does not exceed $\pm 7-8\%$.

Wind speed values, corresponding values of c_p and power P as well as the calculated values of wind motor's rotational speeds ω_{wm} [4] are given in table 1 within the control range of rotational speed.

V, m/s	10,45	9,97	9,44	9,00	8,53	7,97	7,48	7,00	6,51	6,03	5,5	4,96
Р, кW	587,9	535,9	479,2	421,2	362,0	277,4	232,0	186,5	149,3	117,4	86,59	61,6
c _p , r.v.	0,398	0,418	0,441	0,446	0,452	0,423	0,429	0,42	0,418	0,415	0,402	0,39
ω _{wm} rev/s	0,513	0,489	0,463	0,442	0,419	0,391	0,367	0,343	0,319	0,296	0,27	0,243

Value of calculated average value of wind power utilization factor is equal to:

$$c_{p av} \approx 0,42$$

If now to accept within the whole control range $c_p=c_{pav}=const$, then, at determination of power corresponding to maximum c_{pmax} values (in illustrated example it is $P_1 = 362,05 \text{ kW}$ when $c_{pmax} = 0,452$) and minimum c_{pmin} values ($P_2 = 61,6 \text{ kW}$ when $c_{pmin} = 0,39$) the computational error will constitute:

$$\Delta P_{\text{max}} = \frac{P_1 - P_1'}{P_1} = +6,3\%$$
 and $\Delta P_{\text{min}} = \frac{P_2 - P_2'}{P_2} = -8,0\%$,

where $P_1' = 339 \text{ }\kappa\text{W}$ and $P_2' = 66,6 \text{ }\kappa\text{W}$ – are values of power corresponding to wind speeds at c_{pmax} and c_{pmin} determined by formula (1) when $c_p = c_{pav} = 0,42$.

The calculations have shown that when the offered approach of power determination is applied the computational error does not exceed 7–8% for overwhelming majority of WPPs. For instance, for "Vestas V–90" WPP of 2 MW power $c_{p av} = 0,441$, and an error is over the range of from 8% to -2%. Thus, over the range of rotational speed control the WM's power expression can be written in the form of:

$$\mathbf{P}_{wm} = \frac{1}{2} \rho \pi \mathbf{R}^2 \cdot \mathbf{c}_{p \, av} \cdot \mathbf{V}^3 = \mathbf{K}_p \cdot \mathbf{V}^3 , \qquad (7)$$

Table 1

where $K_p = \frac{1}{2}\rho\pi R^2 \cdot c_{pav}$ power proportionality factor, dependent on WM constructive parameters and air mass density, but not dependent on wind speed.

Accordingly the WM torque expression will be:

$$M_{wm} = K_p \frac{V^3}{\omega_{wm}}.$$
 (8)

At regulation specific speed Z remains constant within the whole control range $Z=Z_{opt} = const$ (for adduced WPP $Z_{opt} = 8,0$), so it can be written:

$$V = \frac{\omega_{\rm wm} R}{Z_{\rm opt}} = \frac{R}{Z_{\rm opt}} \cdot \omega_{\rm wm} \,. \tag{9}$$

Substituting the expression (9) into (8) we shall obtain:

$$\mathbf{M}_{wm} = \mathbf{K}_{p} \frac{\mathbf{R}^{3}}{\mathbf{Z}_{opt}^{3}} \cdot \boldsymbol{\omega}_{wm}^{2} = \mathbf{K}_{M} \cdot \boldsymbol{\omega}_{wm}^{2}, \qquad (10)$$

where $-K_{M} = \frac{K_{P} \cdot R^{3}}{Z_{opt}^{3}}$ is a torque proportionality factor.

Thus, wind motor's torque, provided rotational speed regulation in proportion to wind speed with above determined error could be considered as dependent on squared rotational speed.

In modern WPPs for stepless control of wind motor's rotational speed, as a rule, the frequency converters, operating jointly with electromechanical converters, are used. If the super-low speed synchronous generators with "Ringgenerator" permanent magnets (produced by "Enercon", "Vensys") or squirrel- cage asynchronous generators (produced by "Siemens Wind Power") are used as electromechanical converters, then in stator circuits of stated machines the frequency converters executed on fully controlled semiconductor elements (IGBT-transistors, or completely controlled GTO- thyristors) with PDM control are installed. But if to use the double-fed asynchronous machines as electromechanical converter, the stated frequency converters are installed in rotor circuits of these machines (WPPs of "Vestas", "Gamesa" "GE Wind Energy" and etc.).

Presentation of a mathematical model (model of state) and study on this model of different operation modes of WPPs, containing asynchronous generators variable-frequency controlled both from stator side, and from rotor side, will allow to take into account the specific character of stated system operation.

The equations of asynchronous machine in cellular-matrix form are presented in the form of [5]:

$$\begin{bmatrix} \mathbf{p}\boldsymbol{\psi}_{s} \\ \mathbf{p}\boldsymbol{\psi}_{r} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{s1} & \mathbf{A}_{s2} \\ \mathbf{B}_{r1} & \mathbf{B}_{r2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi}_{s} \\ \boldsymbol{\psi}_{r} \end{bmatrix} + \begin{bmatrix} \mathbf{U}_{s} \\ \mathbf{U}_{r} \end{bmatrix},$$
(11)

where the submatrixes expressions assume the form:

$$\mathbf{p}\boldsymbol{\psi}_{s} = \begin{bmatrix} \mathbf{p}\boldsymbol{\psi}_{s\alpha} \\ \mathbf{p}\boldsymbol{\psi}_{s\beta} \end{bmatrix}; \ \mathbf{p}\boldsymbol{\psi}_{r} = \begin{bmatrix} \mathbf{p}\boldsymbol{\psi}_{r\alpha} \\ \mathbf{p}\boldsymbol{\psi}_{r\beta} \end{bmatrix}; \ \mathbf{\psi}_{s} = \begin{bmatrix} \boldsymbol{\psi}_{s\alpha} \\ \boldsymbol{\psi}_{s\beta} \end{bmatrix}; \ \mathbf{\psi}_{r} = \begin{bmatrix} \boldsymbol{\psi}_{r\alpha} \\ \boldsymbol{\psi}_{r\beta} \end{bmatrix}; \ \mathbf{U}_{s} = \begin{bmatrix} \mathbf{U}_{s\alpha} \\ \mathbf{U}_{s\beta} \end{bmatrix}; \ \mathbf{U}_{r} = \begin{bmatrix} \mathbf{U}_{r\alpha} \\ \mathbf{U}_{r\beta} \end{bmatrix};$$

As to submatrixes A_{s1} , A_{s2} , B_{r1} , B_{r2} , they depend on a form of asynchronous machine's equation writing. When writing the equation on fixed in space α , β , 0 axes, they are equal to:

$$\mathbf{A}_{s1} = \begin{bmatrix} -\mathbf{r}_{s}\mathbf{k}_{s} & \mathbf{0} \\ \mathbf{0} & -\mathbf{r}_{s}\mathbf{k}_{s} \end{bmatrix}; \mathbf{A}_{s2} = \begin{bmatrix} -\mathbf{r}_{s}\mathbf{k}_{m} & \mathbf{0} \\ \mathbf{0} & -\mathbf{r}_{s}\mathbf{k}_{m} \end{bmatrix}; \mathbf{B}_{r1} = \begin{bmatrix} -\mathbf{r}_{r}\mathbf{k}_{m} & \mathbf{0} \\ \mathbf{0} & -\mathbf{r}_{r}\mathbf{k}_{m} \end{bmatrix}; \mathbf{B}_{r2} = \begin{bmatrix} -\mathbf{r}_{r}\mathbf{k}_{r} & -\mathbf{\omega}_{r} \\ \mathbf{\omega}_{r} & -\mathbf{r}_{r}\mathbf{k}_{r} \end{bmatrix}$$
(12)

In afore-cited equations the system of selected relative units, base values, indications subject to $p = d/d\tau$, $\tau = 314 \cdot t$ (time in radians), conventional factors k_s , k_r , k_m , are defined from the inverse matrix of machine parameters, i.e.:

$$\begin{bmatrix} \mathbf{k}_{s} & \mathbf{0} & \mathbf{k}_{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{k}_{s} & \mathbf{0} & \mathbf{k}_{m} \\ \mathbf{k}_{m} & \mathbf{0} & \mathbf{k}_{r} & \mathbf{0} \\ \mathbf{0} & \mathbf{k}_{m} & \mathbf{0} & \mathbf{k}_{r} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{s} & \mathbf{0} & \mathbf{x}_{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{x}_{s} & \mathbf{0} & \mathbf{x}_{m} \\ \mathbf{x}_{m} & \mathbf{0} & \mathbf{x}_{r} & \mathbf{0} \\ \mathbf{0} & \mathbf{x}_{m} & \mathbf{0} & \mathbf{x}_{r} \end{bmatrix}^{-1}.$$
 (13)

In expressions (12) – (13): r_s , r_r , x_s , x_r , x_m – accordingly resistances and reactance's of stator (s) and rotor (r) circuits, as well as mutual induction resistance (saturated value of this parameter is taken), ω_r -a rotational circular frequency of asynchronous generator's rotor.

Thus, the expressions (10) - (13) with addition of following equations of motion and torques constitute the mathematical model of WPP's asynchronous machine.

$$T_{j}p\omega_{\Gamma} = m_{em} - m_{wm} m_{em} = k_{m}(\psi_{s\alpha} \cdot \psi_{r\beta} - \psi_{s\beta} \cdot \psi_{r\alpha})$$
(14)

In expression (14) Tj – system's inertial constant (of wind motor and generator) [in radians], $-m_{WM}$ – wind motor's torque, determined on expression (10), is reduced, of course, to generator's axis and introduced in chosen system of relative units. Herewith, naturally, ω_{WM} is also reduced to generator's axis subject to the gain factor of gearbox, as well as pair numbers of generator's poles.

If to study the variable-frequency control mode from the stator side of squirrel-cage generator, it is more simply to use the equations of asynchronous generator recorded on axes, fixed in space. In this case it is necessary to operate with submatrixes A_{s1} , A_{s2} , B_{r1} , B_{r2} .

The imitation of quasistationary operation mode of "Siemens Wind Power" type wind power plants with squirrel-cage generators, controlled with the use of frequency converters that feed the stator windings of stated generators, have been carried out on mentioned mathematical models.

The study results within speed control range of wind motor under. Voltage amplitude control wind constant slip ratio β = const are presented in table 2.

Analysis of results shows that under such regulation the law of the academician M.P. Kostenko is confirmed, i.e. $k_{us} = k_{\pi} \cdot k_{fs} \cdot \sqrt{m_{wm}}$ in this case $m_{wm} = |m_{em}|$ and proportionality factor $k_n = 1,33$.

And, finally, in table 3 the study results are shown for voltage control with the constancy of the reactive power consumed by asynchronous machine. Such control was chosen for a case when it becomes necessary to compensate the reactive power consumed by WPP generator with the aid of the static capacitors bank installed in front of the frequency converter. In such a case there is no need to adjust capacities of these condensers to the speed values of wind – the energy carrier. This function undertakes the converter.

For comparative analysis the calculation of variable–frequency control mode of considered asynchronous generator, carried out by analytical method, is adduced.

In steady-state operational mode, the asynchronous generator's torque controlled by frequency change from stator side, will be balanced by wind motor's torque, reduced to generator's axis:

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$$\mathbf{m}_{\rm em}(\mathbf{k}_{\rm u},\mathbf{k}_{\rm f},\boldsymbol{\beta}) = \mathbf{m}_{\rm wm}, \qquad (16)$$

where $m_{em}(k_u, k_f, \beta)$ – an electromagnetic torque of WPP's asynchronous generator what, in general case, depends on k_u – voltage value, k_f – supplying current's frequency and β – relative slide. According to [7, 8] the equation for generator's electromagnetic torque can be presented in the form of:

$$\mathbf{m}_{\rm em} = \frac{\mathbf{A} \cdot \mathbf{k}_{\rm u}^2}{\mathbf{B} + \mathbf{C} \cdot \mathbf{k}_{\rm f}^2 + \mathbf{D} \cdot \mathbf{k}_{\rm f}},\tag{17}$$

where

$$A = -\left[2r_{s} + (b^{2} + c^{2})\frac{s_{n}}{r_{r}} + (d^{2} + e^{2})\frac{r_{r}}{s_{n}}\right]; \quad B = b^{2}\frac{\beta}{r_{r}} + d^{2}\frac{r_{r}}{\beta}; \quad C = c^{2}\frac{\beta}{r_{r}} + e^{2}\frac{r_{r}}{\beta}; \quad D = -2 \cdot r_{s}$$

In one's turn:

$$\mathbf{b} = -\mathbf{r}_{s}\left(1 + \frac{\mathbf{x}_{\sigma r}}{\mathbf{x}_{m}}\right); \ \mathbf{e} = 1 + \frac{\mathbf{x}_{\sigma s}}{\mathbf{x}_{m}}; \ \mathbf{c} = \mathbf{x}_{\sigma s} + \mathbf{x}_{\sigma r}\left(1 + \frac{\mathbf{x}_{\sigma s}}{\mathbf{x}_{m}}\right); \ \mathbf{d} = -\frac{\mathbf{r}_{s}}{\mathbf{x}_{m}}$$

No	V m/s	m _{em} , r.v	0 rv	k r.v	k. rv	n r.v	$\beta = \omega_r - K_{fs}$,
J 12	v, 111/5		w _r , 1.v	K us, 1.V	K _{fs} , 1.v	Pem, 1.v.	r.v.
1	7	-0,303	0,688	0,49	0,67	-0,208	0,018
2	8,75	-0,471	0,858	0,76	0,84	-0,404	0,018
3	9,8	-0,587	0,958	0,96	0,94	-0,562	0,018
4	10,5	-0,663	1,018	1,09	1	-0,675	0,018
5	7,7	-0,367	0,758	0,60	0,74	-0,278	0,018
6	9,1	-0,505	0,888	0,82	0,87	-0,448	0,018
7	5,25	-0,172	0,518	0,27	0,50	-0,089	0,018
8	9,1	-0,505	0,888	0,81	0,87	-0,449	0,018
9	8,89	-0,479	0,865	0,78	0,847	-0,414	0,018
10	5,25	-0,172	0,518	0,27	0,50	-0,089	0,018
11	4,9	-0,152	0,488	0,24	0,47	-0,074	0,018
12	4,9	-0,152	0,488	0,24	0,47	-0,074	0,018
13	4,9	-0,152	0,488	0,24	0,47	-0,074	0,018
14	7	-0,303	0,688	0,49	0,67	-0,208	0,018
15	8,75	-0,417	0,858	0,76	0,84	-0,404	0,018
16	9,8	-0,587	0,958	0,96	0,94	-0,562	0,018
17	7	-0,303	0,688	0,49	0,67	-0,208	0,018
18	5,25	-0,172	0,518	0,27	0,50	-0,089	0,018
19	7	-0,303	0,688	0,49	0,67	-0,208	0,018
20	5,25	-0,172	0,518	0,27	0,50	-0,089	0,018
21	9,1	-0,505	0,888	0,82	0,87	-0,448	0,018
22	9,1	-0,505	0,888	0,82	0,87	-0,448	0,018
23	8,75	-0,471	0,858	0,76	0,84	-0,404	0,018
24	6,3	-0,244	0,618	0,39	0,60	-0,151	0,018

							Table 3
NG	ν,	m _{em} ,	ω,,	k _{us} ,	k _{fs} ,	p _{em} ,	q ,
JN≌	m/s	r.v.	r.v.	r.v.	r.v.	r.v.	r.v.
1	7	-0,294	0,677	0,750	0,67	-0,199	0,317
2	8,75	-0,465	0,852	0,915	0,84	-0,396	0,317
3	9,8	-0,588	0,958	0,946	0,94	-0,563	0,317
4	10,5	-0,670	1,023	0,960	1	-0,686	0,317
5	7,7	-0,358	0,748	0,870	0,74	-0,268	0,317
6	9,1	-0,50	0,884	0,925	0,87	-0,442	0,317
7	5,25	-0,162	0,503	0,658	0,50	-0,081	0,317
8	9,1	-0,50	0,884	0,925	0,87	-0,442	0,317
9	8,89	-0,473	0,860	0,918	0,847	-0,407	0,317
10	5,25	-0,162	0,503	0,658	0,50	-0,081	0,317
11	4,9	-0,143	0,473	0,625	0,47	-0,068	0,317
12	4,9	-0,143	0,473	0,625	0,47	-0,068	0,317
13	4,9	-0,143	0,473	0,625	0,47	-0,068	0,317
14	7	-0,294	0,677	0,750	0,67	-0,199	0,317
15	8,75	-0,465	0,852	0,915	0,84	-0,396	0,317
16	9,8	-0,588	0,958	0,946	0,94	-0,563	0,317
17	7	-0,294	0,677	0,750	0,67	-0,199	0,317
18	5,25	-0,162	0,503	0,658	0,50	-0,081	0,317
19	7	-0,294	0,677	0,750	0,67	-0,199	0,317
20	5,25	-0,162	0,503	0,658	0,50	-0,081	0,317
21	9,1	-0,50	0,884	0,925	0,87	-0,442	0,317
22	9,1	-0,50	0,884	0,925	0,87	-0,442	0,317
23	8,75	-0,465	0,852	0,915	0,84	-0,396	0,317
24	6,3	-0,238	0,610	0,512	0,60	-0,145	0,317

It is necessary to note, that in general case the parameters of generator rotor's circuit under variable-frequency control depend on relative slide. But under steady- state operational mode the relative slide β , as a rule, must not exceed a value of nominal slide s_n , i.e. $0 < \beta \leq s_n$, so it is possible to consider these parameters constant, which values correspond to $s \approx o$.

It is interesting to note, that neglecting the stator circuit's resistance $r_s=0$, what can be done with asynchronous generator of comparatively greater power, the expression (18) considerably simplifies and takes on the form analogous to the Closs equation for variable- frequency control:

$$m_{em} = -\frac{k_u^2 \left[c^2 \frac{s_n}{r_r} + e^2 \frac{r_r}{s_n}\right]}{k_f^2 \left[c^2 \frac{\beta}{r_r} + e^2 \frac{r_r}{\beta}\right]}$$

The expression (10) determined above for the wind motor torque reduced to the generator axes takes on the form:

$$\mathbf{m}_{\rm wm} = \mathbf{k}_{\rm M\Gamma} \cdot \boldsymbol{\omega}^2 \,, \tag{19}$$

where:

$$\mathbf{m}_{wm} = \frac{\mathbf{M}_{wm}}{\mathbf{i} \cdot \mathbf{M}_{n}}; \ \boldsymbol{\omega} = \frac{\boldsymbol{\omega}_{wm} \cdot \mathbf{i}}{\boldsymbol{\omega}_{0}}; \ \mathbf{k}_{M\Gamma} = \frac{\boldsymbol{\omega}_{0}^{2} \cdot \mathbf{k}_{M}}{\mathbf{i}^{3} \cdot \mathbf{M}_{n}};$$

 M_n, ω_0 – a nominal torque and synchronous speed of generator's rotor; i – gear-ration of the reducer.

Substituting the expressions (17) and (19) in correlation (16) and meaning, that $\omega = k_f + \beta$, we shall obtain:

$$\frac{\mathbf{k}_{u}^{2} \cdot \mathbf{E}}{\mathbf{k}_{f}^{2} \cdot \mathbf{F}} = \mathbf{k}_{M\Gamma} \left(\beta + \mathbf{k}_{f} \right)^{2}, \qquad (20)$$

where $E = c^2 \frac{s_n}{r_r} + e^2 \frac{r_r}{s_n}$; $F = c^2 \frac{\beta}{r_r} + e^2 \frac{r_r}{\beta}$.

At regulation for the constancy of relative slide, i. e. $\beta = \beta_{rel} = const$ within the whole control range we shall obtain the expression for the voltage adjustment:

$$\mathbf{k}_{u} = \mathbf{k}_{f} \sqrt{\frac{\mathbf{k}_{M\Gamma} \cdot \mathbf{E}}{F}} \mathbf{k}_{f}^{2} + \frac{2\mathbf{k}_{M\Gamma} \cdot \mathbf{E} \cdot \boldsymbol{\beta}_{opt}}{F} \mathbf{k}_{f} + \frac{\mathbf{k}_{M\Gamma} \cdot \mathbf{E}}{F} \boldsymbol{\beta}_{opt}^{2} \quad , \tag{21}$$

Taking into account the stator winding's resistance, i. e. for $r_s \neq 0$, the expression becomes complicated and takes on the form:

$$\mathbf{k}_{u} = \sqrt{\frac{\mathbf{k}_{M\Gamma}}{A}} \sqrt{\mathbf{C} \cdot \mathbf{k}_{f}^{4} + \mathbf{N} \cdot \mathbf{k}_{f}^{3} + \mathbf{L} \cdot \mathbf{k}_{f}^{2} + \mathbf{I} \cdot \mathbf{k}_{f} + \mathbf{B} \cdot \boldsymbol{\beta}_{opt}^{2}}, \qquad (22)$$

where $N = D + 2\beta_{opt} \cdot C$; $L = B + 2\beta_{opt} \cdot D + \beta_{opt}^2 \cdot C$; $I = 2\beta_{opt} \cdot B + \beta_{opt}^2 \cdot D$.

We shall compare the results of optimum diagram's calculation, obtained by formula (22), with the ones of calculation of steady-state mode on full algebra-differential equations of WPP's asynchronous generator given in table 3.

The generator's parameters in relative units are: $r_s = 0.022$; $r_r = 0.031$, $x_{\sigma s} = 0.078$, $x_{\sigma r} = 0.1$, $x_m = 4.3$; $s_\mu = 0.035$; $x_s = 4.378$; $x_r = 4.4$; $k_{M\Gamma} = 0.64$; $\omega_0 = 157$ 1/s; $T_j = 100$ rad; $\beta_{opt} = 0.018$; $k_p = 0.171$; i = 33; $M_n = 690$ Nm, $P_n = 110$ kW.

The values of calculated factors are: b = 0,0225; c = 0,1798; e = 1,018; d = 0,0051; A = 0,9989; B = 0,0003387; C = 1,8; D = 0,044; N = 0,1088; L = 0,0025; I = 0,00026.

The final formula for calculations, after simplifying, assumes the form:

$$k_u = k_f \sqrt{1,15 \cdot k_f^2 + 0,0438 \cdot k_f}$$
 (23)

The $k_u = f(k_f)$ curves, plotted by formula (23) (points) and done on the base of values from table 2 (daggers) are presented on fig.1. It is seen from comparison that table 2 data are practically located on $k_u=f(k_f)$ curve plotted by formula (23).



Fig. 1. Dependency $k_u = f(k_f)$ at control with $\beta_{opt} = const$:

- -•- according to offered analytical method;
- -x- according to full algebra-differential equations under steady- state mode

On fig. 2 the WPP asynchronous generator's dynamic characteristics for start-up by underfrequency relay are presented.

In this case the setpoint adjusters define the speeds of grow of amplitude and frequency of the voltage supplied to the stator. Creating wind motor's motive moment the wind relieves the aggregate's acceleration, which is realized by program linear changes of frequency and voltage; upon reaching the synchronous speed, the machine goes over to generator operation with $m_{em} = -0.144$ and $\omega_r = 0.475$ (fig. 2a). Programmed changes of the amplitude and the frequency of the voltage supplied to the generator's stator are illustrated on fig. 2,b.





Fig. 2. a) – the transient curves $m_{em} = f(\tau)$ and $\omega_c = f(\tau)$ for start-up by underfrequency relay; b) – the program linear variations of amplitude $k_{us} = k_{u0} + a \cdot \tau = 0,2 + 0,00135 \cdot \tau$ and frequency $k_{fs} = k_{f0} + b \cdot \tau = 0,1 + 0,00185 \cdot \tau$ of feeding voltage.

CONCLUSIONS

- 1. It is determined, that with acceptable in engineering calculations the error (not more than 8%) in zone of rotational speed control in proportion to speed of wind WPP's power carrier, the wind motor's torque can be considered as proportional to the square of rotational speed.
- 2. The matrix form of state equations of WPP's asynchronous machine under variable- frequency control both from the stator and from the rotor sides allowing to obtain unified results of studies is presented.
- 3. Under variable-frequency control of WPP's asynchronous generator from the stator side the operation modes provided voltage control both on generator's relative slide constancy and on consumed reactive power constancy have been investigated. It was revealed, that first mode agrees with the well-known law of variable-frequency control by academician M.P.Kostenko, and the second mode allows to compensate the WPP's reactive power consumption with the help of uncontrolled static capacitors, installed in points of WPP's connection to electric power network.
- 4. It is demonstrated that in quasystationary modes under variable–frequency control from the rotor side the reactive power output to network can be controlled in double–fed asynchronous machines.
- 5. The analytical method of $k_u = f(k_f)$ calculation provided the control is realized at $\beta_{opt} = const$ is offered. The results of optimum diagram calculation are practically coincide with the results, obtained by full algebra-differential equations (the table 3).
- 6. The dynamic WPPs' modes with asynchronous generator under variable-frequency acceleration of the installation have been investigated; it is revealed, that in this case the values of starting torques and currents minimize.

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