ABOUT ESTIMATION OF EROSION AND REFORMATION OF BANK VAULTS OF GROUND CHANALS AND CANALS

SH. GAGOSHIDZE, T. LORDKIPANIDZE

Approximation theory of the motion of the alongshore wave in a canal communicting with sea is reviwed and a method for estimating deformability of its bank vaults is drafted. One of the most characeristic features of the alongshore waves is an increase of their height close to its bank line. This feature in relief is reflected in an accurate solution of Stokes yet relating to just the waves on the bank vaults infinitely going deep into the sea. For the canals with finite depth and closed contour (in particular, for trapeziod canals) the method of interfacing the solutions of the wave equations written in cylindrical and Cartesian rectangular coordinates respectively for the areas confined with the bank vaults and horizontal bottom of the canal is used.

Key words: Ground beds, slope reformation, alongshore waves, closed loop, interfacing method, trapezoid canal.

Introduction

The waves breaking into the river outlets from the sea side as well as wind and ship waves in rather narrow, elongated basins and canals predominantely have an alongshore direction. The most characteristic for such waves is an increase of their height close to the bank line or the opposite, decrease of an amplitude settled at the bank towards large depths. This feature of the alongshore waves is reflected in three accurate but private solutions belonging to Stokes, Kelland and McDonald [1,2]. Stokes considered the transmission of the short waves of the alongshore vault with an arbitrary dip however infinitely going deep into the sea. Two accurate solutions for the waves on stationary water, in the canals with triangular cross-section against the board dipping at vertical line respectively at 45^0 and 60^0 belong to Kelland and McDonald. It is clear that the absence of accurate general solutions for progressive waves in triangular canal with arbitrary dipping slopes (not to mention the canals with trapezoid cross-section) is explained by mathematic complications.

Taking into the account huge practical importance of the problem of studying the wave influence on the bank slopes of the canals and riverbeds, basic results of approximate theory of the transmission of the alongshore waves superpositioned on the surface of stationary water flow in the trapezoid canal with arbitrary dipping bank slopes are briefly given below [3]. Usage of these results is demonstrated on the example of estimation of the stability and deformation of the bank vault of trapezoid ground bed.

1. Approximate Theory of the Alongshore Waves in the Canal

Using standard ways of coordinate transformation, it is easy to get convinced that the well-known equations and boundary conditions of wave disturbances superpositioned on uniform flow and recorded in Cartesian rectangular coordinates [1,4], in cylindrical coordinates illustrated on figure 1, get the following form:

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\alpha^2}+\frac{\partial^2\varphi}{\partial x^2}=0\,; \tag{1}$$

$$\frac{\partial^2 \varphi}{\partial t^2} + U_0^2 \frac{\partial^2 \varphi}{\partial x^2} + 2U_0 \frac{\partial^2 \varphi}{\partial t \partial x} = -g\left(\cos\alpha \frac{\partial \varphi}{\partial r} + \frac{\sin\alpha}{r}\frac{\partial \varphi}{\partial \alpha}\right), \text{ in } r\cos\alpha = h_0; \tag{2}$$

$$\frac{\partial \varphi}{\partial \alpha} = 0, \qquad \text{in } \alpha = \pm\alpha_0, \tag{3}$$

where φ - velocity potential of the wave disturbances; h_0 – maximum water depth in the trapezoid canal; α_0 - angle of dip of the bank vault at vertical line (axis z) passing through its foundation; r – radius-vector acting within the sector confined with e axis z and the bank vault; U_0 – stationary flow rate in the canal (complying with axis x); marks "$\pm$" are taken respectively in right and left triangular canal sectors. Equation (1) represents Laplace (continuity) equation written in cylindrical coordinates; (2) is linear dynamic boundary condition executable at the level of undisturbed free water surface; and (3) is the condition of impermeability of the canal boards.

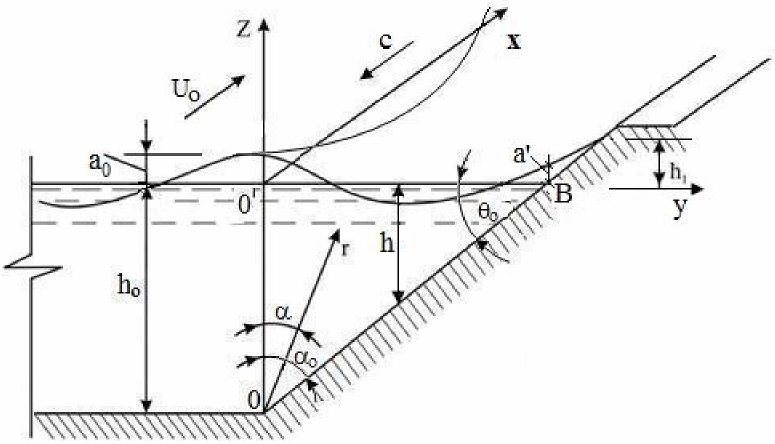


Figure 1. Design Diagram of the Alongshore Waves in Trapezoid Canal

For approximate solution of problem (1)-(3), let's use Kantorovich method [5] representing the potential of the wave disturbance rate in form of the product of function

$$\varphi = f(r)F(\alpha)\exp i(\sigma t \pm kx) \tag{4}$$

and chosen as a base function

$$f(r) = \cosh(kr) \tag{5}$$

in (4) i – imaginary unit; $\sigma = 2\pi/\tau$ - frequency of the wave disturbances; τ - period; $k = 2\pi/\lambda$ – wave number; λ – length of the alongshore wave; marks "$\pm$" correspond with the wave transmission opposite and towards the flow direction.

In accordance with Kantorovich method, putting designations (5) and (4) in (1) and conducting Kalerkin averaging method (1) procedure along the entire turndown r from 0 to $h_0/\cos\alpha_0$, we obtain ordinary differential equation for determining the function $F(\alpha)$:

$$\left(\sinh\frac{2kh_0}{\cos\alpha_0} + \frac{2kh_0}{\cos\alpha_0}\right)F''(\alpha) + \left(\frac{kh_0}{\cos\alpha_0}\cosh\frac{2kh_0}{\cos\alpha_0} - \frac{1}{2}\sinh\frac{2kh_0}{\cos\alpha_0}\right)F(\alpha) = 0. \tag{6}$$

Solution of this equation considering designation (4) and boundary condition (3), gets the following form:

$$F(\alpha) = C\cos m(\alpha \mp \alpha_0), \tag{7}$$

where C – arbitrary constant which is subject to determination or within the rectangular canal part (particularly, over the point O′ provided on figure 1.) or along the bank line (i.e. over the point B).

Following values are identified through m:

$$\mathbf{m} = \pm\left[\frac{\dfrac{kh_0}{\cos\alpha_0}\coth\dfrac{2kh_0}{\cos\alpha_0} - \dfrac{1}{2}}{1 + \dfrac{2kh_0}{\cos\alpha_0}\left(\sinh\dfrac{2kh_0}{\cos\alpha_0}\right)^{-1}}\right]^{1/2}, \tag{8}$$

Configuration of the wave surface and direction orthogonal to the direction of the alongshore waves depends on the above values. In particular, depending on m, crests and troughs of the alongshore waves to the transverse direction of the canal can be solid or may form a chain of standing waves with nodal curves parallel to the bank line. Number of these nodal curves is specified by alternation of function $F(\alpha)$ within the bank vault and it equals to integer part of number n calculated according to formula:

$$\mathbf{n} = \frac{m\alpha_0}{\pi} + \frac{1}{2}, \tag{9}$$

where α_0 expression in radians.

In case of n<1 (i.e. n=0) we are dealing with commensurable and longer alongshore waves compared to the canal depth and width the crests and toughs of which, not intersecting the level of undisturbed water surface, occupy the total canal width.

In case of $n \geq 1$ (i.e. in n=1;2...) the crests and toughs of the alongshore (shorter) waves form the above mentioned wave surface with stationary nodal curves to the transverse direction of the canal (which is illustrated on figure 1).

Limiting length of the alongshore wave according to one case changes into the other is determined by equality

$$\lambda^* = 2\pi h_0 /(0{,}5 + \pi^2 / 4\alpha_0^2)\cos\alpha_0 . \tag{10}$$

Basic complexity of the reviewed problem is to maintain dynamic boundary condition (2) on free flow surface. Without going into the details of the assumptions obtained in [3], will note, that analysis of limit behavior of the dynamic boundary condition (2) in case of $\lambda > \lambda^*$ (n=0) leads to approximate dispersion relation

$$(\sigma - kU_0)^2 = gk\cos\alpha_0 \tanh(kh_0 / \cos\alpha_0), \tag{11}$$

which in the second case, i.e. in case of $\lambda \leq \lambda^*$ (or $n \geq 1$), turns into Stokes dispersion relation for short edge waves [1,2]

$$(\sigma - kU_0)^2 = gk\cos\alpha_0 . \tag{12}$$

In case of n=0, the arbitrary constant C should be normalized through amplitude a_0 specified over the bottom of the bank vault with the requirement that the three-dimensional solutions in the triangular canal part smoothly grow into the well-known solutions for two-dimensional waves in the rectangular canal part. Then, considering the designations (4), (5) and (7) for the velocity potential at the right canal bank will obtain

$$\varphi = a_0 \frac{g}{\sigma - kU_0} \frac{ch(kr)}{ch(kh_0)} \frac{\cos m(\alpha - \alpha_0)}{\cos(m\alpha_0)} \cos(\sigma t \pm kx). \qquad (12)$$

With the same requirement, in case of large n, it is reasonable to normalize constant C (although this is not principal) through the amplitude a' specified along the bank line (described in accepted coordinates by equation $r = h_0 / \cos\alpha_0$). Then, for φ will obtain

$$\varphi = a' \frac{g}{\sigma - kU_0} \frac{ch(kr)}{ch(kh_0 / \cos m\alpha_0)} \cos m(\alpha - \alpha_0) \cos(\sigma t \pm kx). \qquad (13)$$

In all cases, connection between amplitudes a' and a_0 is expressed by dependence

$$\frac{a'}{a_0} = \left| \frac{\cosh(kh_0 / \cos m\alpha_0)}{\cosh kh_0 \cos m\alpha_0} \right|, \qquad (14)$$

according to which a' is always more than a_0 and significantly exceeds it in large kh_0, i.e. in case of presence of the short alongshore waves. At the same time, the alongshore waves moisten the bank slope to the level approximately determined by equality $h_1 = a'/\cos\alpha_0$ (figure 1).

In future, for estimating the deformation of the ground channels, we will need the value of velocity and pressure components of the alongshore waves directly within the plane of the bank slope on which r gets the value $r = (h_0 - h)/\cos\alpha_0$, where h – variable depth of flow over the bank vault (figure 1). Given the above mentioned solutions, will have:

$$u = U_0 \mp AG \sin(\sigma t \pm k x); \qquad (15)$$

$$v = AG \cos(\sigma t \pm k x); \qquad (16)$$

$$p = \gamma h + \gamma A \sin(\sigma t \pm k x) - \gamma \frac{U_0^2}{2g}, \qquad (17)$$

where u and ν - components of the speed of motion of water particles at arbitrary plane point of the bank vault directed towards this plane respectively along the bank line and orthogonal to it; p – pressure; γ - specific gravity of water; upper marks answer for the wave direction opposite and the lower ones – towards the flow of the main flow in the canal with the velocity U_0; A and G – geometric and frequency components of variable wave amplitude calculated when $n<1$ according to dependences

$$A = a_0 \frac{\cosh[k(h_0 - h)/\cos\alpha_0]}{\cosh kh_0 \cos m\alpha_0}; \qquad (18)$$

$$G = \left(\frac{gk}{\cos\alpha_0} \mathrm{cth} \frac{kh_0}{\cos\alpha_0} \right)^{1/2}, \qquad (19)$$

and when $n \geq 1$ – according to Stokes dependance

$$A = a' \exp(-kh/\sin\theta_0); \qquad (20)$$

$$G = (gk / \sin\theta_0)^{1/2}, \qquad (21)$$

where θ_0 – angle between undisturbed water surface and bank vault plane.

These dependences can be the basis for estimating the stability and deformability of unfixed and fixed beds of rivers and canals that are prone to wave influence to the alongshore direction.

2. Design Dependences for Evaluating the Stability and Deformability of the Canal Bank Vaults

Let's demonstrate the above mentioned on private example of the estimation of the stability and deformability of the vaults of trapezoid canal communicating with the sea. The above relations allow to review static stability of soil particles or elements of the protective measures of the canal boards in three directions which are most dangerous in terms of erosion:

a) upwards, towards the direction normal to the bank vault plane when the wave bottom passes over the element laying on the slope, and a hydraulic uplift formed by averaged (zero) water level in the canal affects from below;
b) towards the flow direction conditioned by the flow of the soil particles or stones of the protecting fill by both the main flow and the alongshore waves;
c) downwards, towards the slope plane, perpendicular to the bank line direction conditioned by three-dimensional structure of the alongshore waves.

Consideration of static stability of the soil particles laying on the bank vault according to these three directions respectively leads to the following equations:

$$\gamma_s^{\bullet} d^3 \cos\theta_0 + c_o d^2 + \gamma A d^2 \sin\beta = 0 \;; \tag{22}$$

$$\gamma_s^{\bullet} f d^3 \cos\theta_0 + c_o d^2 - \frac{1}{2}\overline{C}\rho d^2 (U_0 + AG)(U_0 \pm AG\sin\beta) = 0\;; \tag{23}$$

$$\gamma_s^{\bullet}(f\cos\theta_0 - \sin\theta_0)d^3 + c_o d^2 - \frac{1}{2}\overline{C}\rho d^2 (U_0 + AG)AG\cos\beta + f\gamma A|\sin\beta| d^2 = 0, \tag{24}$$

where d – soil particle size reduced to cube; $\gamma_s^{\bullet} = (\gamma_s - \gamma)$ - soil specific gravity in suspension; γ and ρ – specific gravity and density of water; f – friction coefficient ($f \approx 0.7$); c_0 – coefficient of soil particle coalescence; $\overline{C}$ – coefficient of head resistance ($\overline{C} \approx 1.05$ [86]); A and G – constituent wave amplitudes calculated according to the formulas (18)-(19) or (20)-(21) depending on the value of n number determined according to the equality (9); $\beta = (\sigma t - kx)$ – wave phase passing over the particle. In accordance with (22), the most dangerous value of the phase $\beta = -\pi/2$ for the particle displacement corresponds with the passage of the wave bottom over the particle; and in accordance with (23) – passage of the wave crest ($\beta = \pi/2$), if the wave direction coincides with the direction of the stationary flow or with the passage of the wave crest ($\beta = -\pi/2$) otherwise. In (24) experimental value of the wave phase β depends on the velocity U_0 and the wave amplitude AG. In the first approximation, it is possible to accept $\beta \approx n\pi$.

Determining size d of extremely stable soil particles towards the above three directions, we chose the largest one as a design quantity.

It should be noted, that together with increasing the flow depth h at the bank vault ($0 \le h \le h_0$), eroding ability of the alongshore waves decreases significantly. For instance, assuming that the short waves are transmitted along the canal, and proceeding from the relations (22) and (20), then such decrease occurs according to exponential law. From these relations, the limiting flow depth (h_w) in which the force of the hydraulic uplift is balanced by the force of the particle of incoherent soil (c_0=0) laying on the bank vault equals to

$$h_w = \frac{\lambda}{2\pi\sqrt{1+m_0^2}}\,\ell n\left(\frac{\gamma}{\gamma_s^{\bullet}}\frac{a'}{d}\frac{\sqrt{1+m_0^2}}{m_0}\right), \tag{25}$$

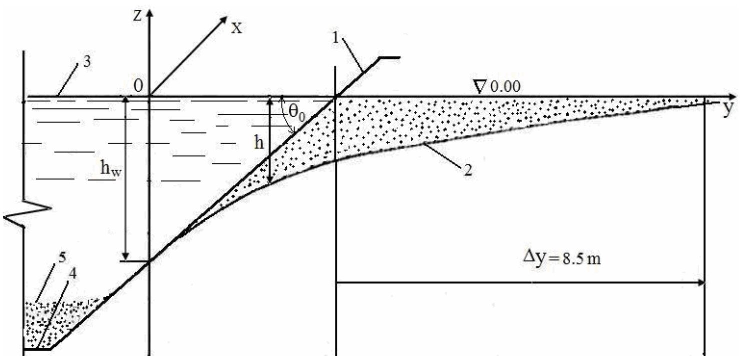
where a' – specified or defined amplitude of the alongshore wave under formula (14); λ– wave length; $\mathbf{m_0 = ctg\theta_0}$– initial (design) value of the bank slope coefficient.

Relation (25) can be the basis for designing differential equation of the contour line of the eroded bank vault above the point defined by the depth h_w. For this, let's shift parallel coordinate system illustrated on figure 1 to the point under which the water depth h_w and in (25) replace h_w and m_0 respectively by variables h=f(y) and $\mathbf{m'_0 = -\frac{dy}{dh}}$. Let's accept that during erosion, the wave amplitude and length remain the same and the value $\sqrt{\mathbf{1 + m'^2_0}}$ gets an approximate value $\sqrt{\mathbf{1 + m'^2_0}} \approx \mathbf{m'_0}$ (considering that usually the slope coefficient m'_0 at the eroded bank is several times more than one and hence, $\mathbf{m'^2_0 >> 1}$). Than, taking into the account these assumptions, (25) turns to the linear differential equation

$$\mathbf{h = -\frac{\lambda}{2\pi}\frac{dh}{dy}\ell n\left(\frac{\gamma}{\gamma^\bullet_s}\frac{a'}{d}\right)}. \tag{26}$$

Solution (26) in observing the boundary condition h = h_w when y=0 determines the water depth over the eroded vault or the outline of the deformated bank vault, which is the same.

Calculation results according to formulas (25) and (26) of the reformation of one really acting ground channel communicating with the marine harbor and functioning for transporting barge and other port vessels are provided on the below given figure 2. Within two years since putting the wave canal formed by the barges into the operation caused strong distortion of its original trapezoid cross-section eroding and sliding the banks by more than 8 m. The canal vaults became significantly gentle and due to soil silting withdrawn from the canal banks their bottom level elevated threatening navigation.



y, m	0	2	4	4.5	6	8	10	12	13
h_w, m	1.28	0.71		0.00					
h, m	1.28	0.83		0.49	0.35	0.29	0.15	0.11	0.07
m'_0	3.5	5.6		9.5	13.3	16.0	30.9	46.4	66.3

Figure 2. Calculation Results of Deformation of the Bank Vault of the Canal:
1 – design contour of the bank vault; 2 – contour of the eroded vault; 3 – undisturbed flow surface in the canal; 4 – original bottom of the canal; 5 – soil deposited on the canal bottom; h_w – design water depth on the vault; e – transverse coordinate calculated from vertical line where h_w = 1,28m; h – water depth on the eroded vault; m'_0 - coefficient of slope of the eroded bank vault.

Calculations were run with the following basic data: initial (design) value of the coefficient of slope $\mathbf{m_0 = ctg\theta_0}$=3,5; average size of soil grains of the canal vaults $\mathbf{d = 0,5 \cdot 10^{-4}}$ м; specific gravity of water and soil particles under suspension $\gamma = 1$ t/m^3 and $\gamma_s^\bullet$= 1,6 t/m^3; length and amplitude of the alongshore wave λ= 4m and a'= 0,35m. Calculation results appeared to be in good compliance with the data of in-situ observations on the erosion of the coasts if sea port canals.

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Shalva Gagoshidze, Doctor of Technical Sciences, Professor,
Georgian Research Institute of Power Engineering and Power Structures,
0171, Georgia, str. Kostava, 70
Tel.: (+995 32) 226260; mob.: +995 893 235293
E-mail: sh.gagoshidze@gmail.com