# OPTIMAL DISTRIBUTION OF ACTIVE LOAD IN POWER SYSTEM CONSIDERING THE SERIES OF HYDROPOWER PLANTS

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*Optimal distribution of the active load in the power system considering the series of hydropower plants is reviewed.*
*Efficiency function, system of the constraint equations and inequalities were drafted based on mathematic expressions of the plant metering characteristics. Lagrange indefinite multiplier method was applied in the problem solution.*
*An equation system for the problem of the active load distribution optimization between the power plants in the power system taking into the account the seasonal hydropower plants operating in series with the hydropower plants with reservoirs the operation of which is somehow associated with the operating condition of regulating hydropower plants is obtained.*

**Key words: constraint equations, minimization of fuel consumption, function extremum, active load, balancing thermal plant.**

**The equation system of the optimization problem connects the engineering-and-economical performance of the production and technological process to the main operation parameters of the unit to be researched. These parameters are the sought parameters of the problem.**

**Mathematic model for the optimization of any technological process generally contains five equation groups: effectiveness i.e. aim equation, coupling equation, constraint equations and inequalities, optimal management equation, adaptation equation.**

**Optimal distribution of the active load between hydropower plants is one of the main activities in the power system helping to achieve as small consumption of the primary energy resource as possible. In the power system the problem of minimizing the consumption of the primary energy resource under the conditions of limited (preliminarily determined) water resource arrives to the problem of minimizing the fuel consumption at the thermal plants. In such case, the efficiency function has the following form [1-3]:**

$$T = \sum_{t=1}^{k} \sum_{i=0}^{n} T_{i,t} \tau_t => min, \quad (1)$$

**Where $\tau_t$ is the time interval within which the total power system load can be considered as a constant value (for simplicity $\tau_t = 1$hr is usually taken); k - $\tau_t$ is the total amount of the intervals within the involved period; $T_{i,t}$ – i is an hourly fuel consumption in $\tau_t$ interval at the thermal plants; 0 – index is granted to balancing thermal plant; n – total number of the remainder thermal plants.**

**Coupling equations are presented by metering characteristics of the power plants in form of second row polynomial [2]:**

$$T_{it} = T_{o,t} + a_i P_{i,t} + b_i P_{i,t}^2 \text{ (kg/h); } i = 0,1,2,...n \quad (2)$$

**and**

$$Q_{j,t} = Q_{o,j} + a_j P_{j,t} + b_j P_{j,t}^2 \text{ (m}^3\text{/sec); } j = \alpha,\beta,\mu. \quad (3)$$

**Here, $T_{i,t}$ - i is an hourly fuel consumption at the thermal plant; $Q_{j,t}$ – i is the total water consumption at the hydropower plant; $P_{i,t}$ – load of the thermal plants at $\tau_t$ interval; $P_{j,t}$ - load of the hydropower plants at $\tau_t$ interval; $T_{0,t}$, $Q_{0,j}$, $a_i$, $b_i$, $a_j$, $b_j$ – polynomial coefficients; $\mu$ - total number of the hydropower plants.**

**The following is considered in form of the constraint equations:**

- **active capacity balance equation for each $\tau_t$ interval**

$$W_{p,t} = \sum_{i=0}^{n} P_{i,t} + \sum_{j=\alpha}^{m} P_{j,t} + \sum P_{\text{remainder plants},t} - P_{\text{load},t} = 0, \qquad (4)$$

**Where $P_{\text{remainder plants } t}$ – total load of the remainder power plants operating in the system including the seasonal hydropower plants operating in series with regulating hydropower plants the operating condition of which is somehow associated with the operating condition of the regulating hydropower plants; m – total number m<μ of regulating (having reservoirs) hydropower plants;**

- **daily water limit for particular regulating hydropower plants**

$$W_{Q,j} = 3600 \cdot \sum_{t=1}^{24} Q_{j,t}\tau_t - Q_{j,\text{lim}} = 0 \quad j = \alpha, \beta \ldots m \qquad (5)$$

**Where, $Q_{j,\text{lim.}}$ j is daily water limit ($m^3/d$) for the hydropower plant.**

**As a coupling inequality we have:**

$$P_{i,\min} \le P_i \le P_{i,\max} \text{ and } P_{j,\min} \le P_j \le P_{j,\max}. \qquad (6)$$

**Optimal management, i.e. optimization equation allows to conduct the optimal process management. It is obtained by joint consideration of the aim, coupling and constraint equations and is the link between the sought variables and the aim of the optimization problem.**

**In the solution of the optimization problems Lagrange indefinite multiplier method is used. Here, instead of the efficiency function extremum written in (1) form the Lagrange function extremum is considered and written in a following form:**

$$L = T + \lambda_t W_{p,t} + \sum_{j=\alpha}^{m} \lambda_j W_{Q,j} => min,$$

**i.e.**

$$\begin{aligned} L = {} & \Sigma_{t=1}^{24}\Sigma_{i=0}^{n}(T_{0,i,t} + a_i P_{i,t} + b_i P_{i,t}^2 + \\ & + \Sigma_{i=1}^{24}\lambda_t(\Sigma_{i=0}^{n} P_{i,t} + \Sigma P_{dan.sadg.t} - \\ & - P_t - \Delta P) + \Sigma_{j=a}^{m}\lambda_j[3600\Sigma_{t=1}^{24}(Q_{0,j,t} + aP_{j,t} + b_j P_{j,t}^2) - Q_{j,lim}] => \min. \end{aligned} \qquad (7)$$

**In this expression $\tau_t$=1, and $\lambda_t$, $\lambda_i$ are the multiplication of the Lagrange indefinite multipliers.**

**By differentiating the Lagrange function (7) with $P_i$ and $P_j$ will get:**

$$\left.\begin{aligned} \frac{\partial L}{\partial P_{0,t}} &= \varepsilon_{0,t} + \lambda_t\left(1 - \frac{\partial \Delta P}{\partial \Delta P_{0,t}}\right) = 0 \\ \frac{\partial L}{\partial P_{1,t}} &= \varepsilon_{1,t} + \lambda_t\left(1 - \frac{\partial \Delta P}{\partial \Delta P_{1,t}}\right) = 0 \\ & - - - - - - - - - - - - \\ \frac{\partial L}{\partial P_{i,t}} &= \varepsilon_{i,t} + \lambda_t\left(1 - \frac{\partial \Delta P}{\partial \Delta P_{i,t}}\right) = 0 \\ & - - - - - - - - - - - - \\ \frac{\partial L}{\partial P_{n,t}} &= \varepsilon_{n,t} + \lambda_t\left(1 - \frac{\partial \Delta P}{\partial \Delta P_{n,t}}\right) = 0 \end{aligned}\right\} \qquad (8)$$

**and**

$$\left.\begin{array}{l}\frac{\partial L}{\partial P_{\alpha,t}} = \lambda_t\left(1-\frac{\partial\Delta P}{\partial\Delta P_{\alpha t}}\right)+3600\lambda_{\alpha,t}\varepsilon_{\alpha,t} = 0\\ \frac{\partial L}{\partial P_{\beta,t}} = \lambda_t\left(1-\frac{\partial\Delta P}{\partial\Delta P_{\beta t}}\right)+3600\lambda_{\beta,t}\varepsilon_{\beta,t} = 0\\ ------------\\ \frac{\partial L}{\partial P_{j,t}} = \lambda_t\left(1-\frac{\partial\Delta P}{\partial\Delta P_{jt}}\right)+3600\lambda_{j,t}\varepsilon_{j,t} = 0\\ ------------\\ \frac{\partial L}{\partial m,t} = \lambda_t\left(1-\frac{\partial\Delta P}{\partial\Delta P_{\mu t}}\right)+3600\lambda_{m,t}\varepsilon_{m,t} = 0\end{array}\right\} \quad (9)$$

**According to the sought variables $P_{i,t}$ and $P_{j,t}$ the function minimization requirement provides the following equation system:**

$$\varepsilon_{i,t} + \lambda_t = 0 \qquad i = 1,2 \dots n$$
$$\lambda_t + 3600\lambda_j\varepsilon_{j,t} = 0 \qquad j = \alpha \dots m.$$

**After excluding $\lambda_t$ indefinite multiplier in the derived equation system, will get that the necessary condition for the optimal distribution of the active load between the plants is the equality of relative increments of their primary energy resource.**

$$\varepsilon_{1,t} = \varepsilon_{2,t} = \dots\varepsilon_{n,t} = 3600\lambda_\alpha\varepsilon_{\alpha,t} = 3600\lambda_\beta\varepsilon_{\beta,t} = \dots = 3600\lambda_m\varepsilon_{m,t} = \varepsilon_0 \quad (10)$$

**i.e.**

$$\left.\begin{array}{c}\varepsilon_1 = \varepsilon_0\\ \varepsilon_2 = \varepsilon_0\\ ---\\ \varepsilon_n = \varepsilon_0\\ 3600\lambda_\alpha\,\varepsilon_\alpha = \varepsilon_0\\ ----\\ 3600\lambda_m\,\varepsilon_m = \varepsilon_0\end{array}\right\}.$$

**(4), (5) and (10) equations jointly represent the equation system, which in form of (6) considering the inequality allows to resolve the set problem.**

**$\lambda_j$ multiplier included in this equation system represents the relation between the fuel consumption at balancing thermal plant and water consumption at $j$ regulating hydropower plant. In particular, $\lambda_j$ shows how (in what quantity) the fuel consumption (kg) at balancing thermal plant changes in case of changing the water consumption at $j$ regulating hydropower plant by one unit (m$^3$). For given hydropower plant it is called the efficiency coefficient of water resource.**

## REFERENCE


1. მახარაძე გ. ენერგოსისტემების რეჟიმების მართვა და ოპტიმიზაცია. თბილისი:ტექნიკური უნივერსიტეტი. 2005.
2. **Веников В.А. и др. Оптимизация режимов электростанций и энергосистем. Учебник для Вузов. М.:Энергоиздат. 1981.**
3. **Холмский В.Г. Расчет и оптимизация режимов электрических сетей. М.:Высшая школа. 1975.**