## INFLUENCE OF SURFACE WAVES ON THE DISTRIBUTION OF CONTAMINANTS IN COASTAL AREAS OF SEA

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In the present paper, we derive relations which can be used to predict the size of propagation of a polluting impurity cloud or spot in the pre-coastal area of the sea for the purposes of construction of sewage water outlets and clean sea water inlets in the recreation zones. In paragraph 2 we consider the problem of a conservative impurity flow from the point source towards the sea shore (the problem of a polluting water outlet) taking into account the river flow and the surge of waves over the shore. The third paragraph deals with the propagation of a polluting impurity from the shore (a linear source) towards the sea where the clean water intake is located. As different from the first problem considered in the second paragraph, in this problem the diffusion turbulence coefficient is determined taking into account longitudinal coastal waves and drifting flows caused by wind. At the end of the paper, we give some numerical results on the impurity concentration distribution calculated by the obtained relations for the clean sea water intake structure which is being designed in one of the resort areas of Georgia's Black sea coast.

**Key words:** surface waves, pollution, impurity distribution, incompressible stream, impurity concentration, diffusion coefficients.

1. Basic equations of the impurity distribution in the pre-coastal zone. As is known, the initial three-dimensional equation of the turbulent-advective transfer of polluting impurities by an incompressible stream has the form [1, 2, 3]

$$\frac{\partial C}{\partial t} + V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y} + V_z \frac{\partial C}{\partial z} = \frac{\partial}{\partial x} \left( D_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( D_z \frac{\partial C}{\partial z} \right) - kC , \qquad (1)$$

where C is the impurity concentration,  $V_x$ ,  $V_y$ ,  $V_z$  are the flow velocity components;  $D_x$ ,  $D_y$ ,  $D_z$  are the diffusion coefficients along the respective axes; k is the impurity non-conservativity coefficient.

Equation (1) describes the averaged over a certain time interval t field of impurity in the turbulized water, but it is too complicated for the application and its analytical solution can be obtained if the size of the source is not large, its discharge and action time have small values [3]. As to problems with a permanently acting pollution source and flows of different nature similar to the flows to be considered below, their solution leads to a further stylization of equation (1) In particular, using the Boussinesq averaging method and also making some assumptions for the integral coefficients of turbulent diffusion, conservativity and the lateral outflow of polluting impurities, equation (1) is reduced to the following one-dimensional (hydraulic) equation

$$\frac{\partial \mathbf{C}}{\partial \mathbf{t}} + \mathbf{V}_{\mathbf{x}} \frac{\partial \mathbf{C}}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}} \left( \mathbf{D}_{\mathbf{x}} \frac{\partial \mathbf{C}}{\partial \mathbf{x}} \right) - \mathbf{C} \mathbf{k}_{\mathbf{f}}$$
(2)

where  $V_x$  an average flow velocity over a free cross-sectional area; c is the impurity concentration averaged over the cross-sectional area;  $D_f$  is the total value of the molecular and turbulent diffusion coefficients along the orthogonal x-, y-, z-axes;

$$D_f = D_{xm} + D_{xt} + (D_y)_{xturb} + (D_z)_{xturb}$$
(3)

is the integral coefficient of turbulent diffusion;

$$k_f = k_I + k_s \tag{4}$$

is the coefficient of impurity non-conservativity taking into account the lateral (along the main *x*-direction) impurity inflow or outflow.

Depending on the character of the problems we will consider below, equation (2) can be simplified even more. The initial and boundary conditions which must be satisfied when solving these problems are given in paragraphs 2 and 3.

2. The problem on the permanently acting source of an impurity flow in the case of overlapped pulse excitations. Let the permanently acting sewage water outlet be located at a distance L from the sea shore. It is assume that at the point where the polluting impurity flows upwards its concentration varies relative to the average value of C<sub>0</sub> according to the law C<sub>0</sub>'cos $\beta$ t, where C<sub>0</sub>' is the given amplitude of impurity concentration deviation above the water outlet with the  $\beta$  frequency. Then, assuming that the *x*-axis is directed from the site of water outlet location orthogonally to the sea coastline, the boundary conditions for equation (2) are written in the form

$$C(0,t)=C_0+C_0'\cos\beta t,$$
(5)

$$\mathbf{C}(\mathbf{L},\mathbf{t}) = \mathbf{C}_{\mathbf{n}},\tag{6}$$

where  $C_n$  is the impurity concentration allowed by official rules near the sea coastline.

Below we will consider the problem in its most simplified formulation assuming that the impurity is conservative ( $k_f = 0$ ) and the flow velocity and the diffusion coefficient are constant values equal to  $V = V_0$  and  $D_f = D_0$ , respectively. By the boundary condition (5) we can represent the solution in the following complex form:

$$C(x,t) = \overline{C}(x) + C'(x,t)C^{\beta t}$$
(7)

where  $\overline{C}_x$  is the stationary component of concentration distribution, C'(*x*, *t*) is the concentration excitation amplitude along the *x*-axis. Then, splitting equation (2) into a stationary and a non-stationary part and taking into account the assumptions made above, we define  $\overline{C}$  by the equation

$$V_0 \frac{d\overline{C}}{dx} = D_0 \frac{d^2\overline{C}}{dx^2}$$
(8)

while to define C'(x,t) we have the equation

$$\frac{\partial \mathbf{C'}}{\partial t} + \mathbf{V}_0 \frac{\partial \mathbf{C'}}{\partial \mathbf{x}} = \mathbf{D}_t \frac{\partial^2 \mathbf{C'}}{\partial \mathbf{x}^2} \tag{9}$$

Solving equation (8) with the boundary conditions  $\overline{C}_x = C_0$  for x = 0 and  $\overline{C}_x = C_n$  for x = L, we obtain

Sh.Gagoshidze,....

$$\overline{C}(x) = C_0 - (C_0 - C_n) \frac{e^{\frac{V_0}{D_0}x} - 1}{e^{\frac{V_0}{D_0}L} - 1}$$
(10)

As we see from (10), for any direction of the impurity flow towards the sea the impurity concentration decreases towards the seashore though with a different intensity.

Let us now solve equation (9) which describes steady oscillations of impurity concentration excitations. A solution will be sought in the form

$$\underline{\mathbf{C}}'(\mathbf{x},t) = C_0' \mathrm{e}^{-\lambda \mathrm{x} + \mathrm{i}\beta \mathrm{t}}$$
(11)

Substituting it in equation (9), we obtain the following expression for defining the constant  $\lambda$ :

$$\lambda^2 + \lambda \frac{V_0}{D_0} - i\beta = 0$$
 (12)

which in the case we are interested in, i.e. for large oscillation frequencies of the polluting substance eruption gives

$$\lambda = -\frac{V_0}{2D} + (1+i)\sqrt{\frac{\beta}{2D_0}}.$$
(13)

Then separating the real part, by virtue of (11) we find

$$C'(x,t) = C_0' e^{(\frac{V_0}{2D_0} - \sqrt{\frac{\beta}{2D}})X} \cos(\beta t - \sqrt{\frac{\beta}{2D_0}}x).$$
(14)

It follows from (14) that in the case of a pulsating pollution source, the impurity concentration directed towards the sea shore ( $V_0 < 0$ ) with a delayed phase decreases more rapidly than in the case of a permanently acting non-pulsating source ( $\beta = 0$ ) of the same capacity.

3. The case of propagation of a polluting impurity from the shore inwards the sea. In this case, wind and waves directed towards the shore drive the polluting substances towards the sea coastline and therefore when choosing the location for the construction of a clean sea water intake it is necessary to take into account the mechanism of a diffusive and advective transfer of impurities towards the sea which is caused by longitudinal wave flows. According to these facts, in the considered problem the *y*-axis coincides with the coastline, while the *x*-axis is directed towards the sea, the turbulent diffusion coefficient can be set equal to  $(D_y)_{xturb}$  in expression (3), while in equation (2) it can be written in the form

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} - Ck$$
(15)

This equation does not take into account the term  $V_x \frac{\partial C}{\partial x}$  because of its smallness. Replacing the variable  $C = \overline{C}e^{-kt}$  in (13), we rewrite it in the form

$$\frac{d\overline{C}}{dt} = D_x \frac{d^2\overline{C}}{dx^2},$$
(16)

The boundary and initial conditions for the problem are formulated as follows. The boundary condition is

$$C_{x=0} = C(0,t) = \tilde{C}_0$$
 (17)

Sh.Gagoshidze,....

where  $\tilde{C}_{0}$  is the constant inflow of impurities in the initial stage. The initial condition is

$$C_{t=0} = f(x) = \begin{cases} 0, & 0 < x < x_1 \\ C_0, & x_1 < x < x_2 \\ 0, & x > x_2 \end{cases}$$
(18)

where  $x_2 - x_1$  is the width of the pre-coastal impurity strip.

Using the Fourier integral [4], for equation (16) we obtain

$$C(x,\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} f(\xi) \cos \lambda (x-\xi) e^{-\lambda^2 \tau} d\xi$$
(19)

which satisfies the initial condition  $C|_{t=0} = f(x)$ .

Applying the transformation  $\lambda = \frac{\sigma}{\sqrt{t}}$ , introducing the notation  $\frac{x-\xi}{\sqrt{t}} = \omega$  and

keeping in mind that  $\tau = Dt$ , we obtain

$$C(x,\tau) = \frac{1}{2\sqrt{D\pi t}} \int_{-\infty}^{\infty} f(\xi) e^{\frac{(x-\xi)^2}{4Dt}} d\xi$$
(20)

If we consider the propagation of impurities from the coastal line into the sea depth along the x-axis which coincides with the normal to the coastal line, we will thus satisfy the initial condition (17) and the boundary condition (18). Also, if we continue the function f(x)onto the negative Ox-semi-axis so that it would take an even value, then, after some transformations, we obtain

$$C(\mathbf{x}, \mathbf{t}) = \widetilde{C}_{0} \left\{ 1 - \Phi \left( \frac{\mathbf{x}}{2\sqrt{(\mathbf{D}_{m} + \mathbf{D}_{T})\mathbf{t}}} \right) \right\} + \frac{C_{0}}{2} \left\{ 2\Phi \left( \frac{\mathbf{x}}{2\sqrt{(\mathbf{D}_{m} + \mathbf{D}_{T})\mathbf{t}}} \right) - \Phi \left( \frac{\mathbf{x} - \mathbf{l}}{2\sqrt{(\mathbf{D}_{m} + \mathbf{D}_{T})\mathbf{t}}} \right) - \Phi \left( \frac{\mathbf{x} + \mathbf{l}}{2\sqrt{(\mathbf{D}_{m} + \mathbf{D}_{T})\mathbf{t}}} \right) \right\}$$
(21)

where  $\Phi$  is a Laplace function and it is assumed that  $x_1 = 0$  and  $x_2 = l$ .

To obtain the final solution, we multiply (21) by  $e^{-kt}$ .

The obtained relations were used in the test design calculations for the concrete area of the Black sea coast of Georgia (near the port of Poti). Here the coast is grown with macrophytoalgae in which large quantities of plankton got accumulated, Calculations were performed for actinomycetes (having a highly pungent smell) the smell of which can be perceived even at a concentrayion of the odorous substance equal to

 $10^{-8}$  mg/l. The transverse component of the turbulent diffusion coefficient is calculated according to [5]:

$$(D_y)_{xturb} = 0.7 \cdot 10^{-2} \text{ m}^2/\text{sec} >> D_{mol} \approx 10^{-10} \div 10^{-12} \text{ m}^2/\text{sec}; \text{ k} = 5.78 \cdot 10^{-7} \text{ 1/sec}.$$

Calculations were performed for maximal concentrations achieved after t = 11 hours (the cycle of active exudation of odorous substances).

The results of calculations are presented in the table from which we see that in these conditions, already at a distance of 70 m from the propagation of algae and plankton, the impurity concentration drops from 0.7 mg/l to  $3.8 \cdot 10^{-3} \text{ mg/l}$ , i.e. it decreases 184 times.

Table

<i>x</i> , m	0	5	10	15	30	50	70	80	1
$C_0$ , mg/l	0.7	0.58	0,49	0.38	0.16	0.03	0.0038	0.0012	

Thus the relations obtained in the paper allow us to determine the location of water outlet and water intake facilities.

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